Post-quantum cryptography

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Noisebridge, San Francisco
“In the past, people have said, maybe it’s 50 years away, it’s a dream, maybe it’ll happen sometime. I used to think it was 50. Now I’m thinking like it’s 15 or a little more. It’s within reach. It’s within our lifetime. It’s going to happen.”

—Mark Ketchen (IBM), Feb. 2012, about quantum computers
“Whether we can control the quantum states and all of that at the fundamental level has now been proven. The big killer is, at what point do we build a processor big enough that’s it’s faster than a classical computer?

That means moving away from small scale models to integrated processing devices and prototypes. That’s the challenge, and that can be done, we anticipate, within the next decade.”

—Michelle Simmons (UNSW), Jan. 2016
Why would cryptographers care?

Grover’s algorithm (1996)

- Find preimages of a blackbox function in $O(\sqrt{N})$
- $N$ is the size of the domain of the function
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Shor’s algorithm (1994)

- Factor integers in polynomial time
- Compute discrete logarithms in polynomial time
- Complete break of RSA, ElGamal, DSA, Diffie-Hellman
- Complete break of elliptic-curve variants (ECSDA, ECDH, . . . )
Is public-key crypto dead?
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- Larger keys, signatures, ciphertexts (for some)
- Security less well understood (for some)
- Additional issues (e.g., stateful hash-based signing)
NIST post-quantum crypto project

- NIST issued a (draft) call for PQC proposals
- Submissions for
  - PQ signatures
  - PQ encryption
  - PQ key agreement
- Submission deadline: November 2017
- Submitters’ presentations: Early 2018
- 3–5 years of analysis
- 2 years later: draft standards ready
- See http://csrc.nist.gov/groups/ST/post-quantum-crypto/
PQCRYPTO

- Project funded by EU in Horizon 2020.
- Starting date 1 March 2015, runs for 3 years.
- 11 partners from academia and industry, TU/e is coordinator
- Goal: **Design and implement high-security post-quantum PKC**
NSA’s data center in Bluffdale
Estimated numbers

- Electricity consumption: 65 MW
- Energy bill: US$40,000,000/year
- Storage: 3–12 EB
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The attack scenario
- Store encrypted data now
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The attack scenario

- Store encrypted data now
- Decrypt in 15 (?) years
- Consequence:

  Need post-quantum encryption now!
How about PFS?

“Perfect Forward Secrecy”:

- Use long-term secret keys for authentication only
- Use short-term _ephemeral_ keys for encryption
- Compromise of long-term key does not compromise confidentiality of past messages
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*Does not help* against cryptanalytic break

Attacker breaks (in poly time) each single ephemeral key exchange
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- *Does not help* against cryptanalytic break

- Attacker breaks (in poly time) each single ephemeral key exchange

- As a consequence, we want
  - *ephemeral key exchange* (to protect against key compromise)
  - *post-quantum security* (to protect against future quantum attacker)
POST-QUANTUM KEY EXCHANGE

A NEW HOPE

ERDEM ALKIM
LÉO DUCAS
THOMAS PÖPEL accordion
PETER SCHWABE
Ring-Learning-with-errors (RLWE)

- Let $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n + 1)$
- Let $\chi$ be an error distribution on $\mathcal{R}_q$
- Let $s \in \mathcal{R}_q$ be secret
- Attacker is given pairs $(a, as + e)$ with
  - $a$ uniformly random from $\mathcal{R}_q$
  - $e$ sampled from $\chi$
- Task for the attacker: find $s$
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- Common choice for $\chi$: discrete Gaussian
- Common optimization for protocols: fix $a$
A bit of (R)LWE history

- Regev, 2005: Introduce LWE-based encryption
- Lyubashevsky, Peikert, Regev, 2010: Ring-LWE and Ring-LWE encryption
- Ding, Xie, Lin, 2012: Transform to (R)LWE-based key exchange
- Peikert, 2014: Improved RLWE-based key exchange
- Bos, Costello, Naehrig, Stebila, 2015: Instantiate and implement Peikert’s KEX in TLS
**Peikert’s RLWE-based KEM**

<table>
<thead>
<tr>
<th>Parameters:</th>
<th>$q, n, \chi$</th>
</tr>
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<tbody>
<tr>
<td><strong>KEM.Setup()</strong> :</td>
<td></td>
</tr>
<tr>
<td>$a \leftarrow_R q$</td>
<td></td>
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<tr>
<td><strong>Alice (server)</strong></td>
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<td><strong>KEM.Gen(a)</strong> :</td>
<td><strong>KEM.Encaps(a, b)</strong> :</td>
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<td>$s, e \leftarrow \chi$</td>
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<td>$u \leftarrow as' + e'$</td>
</tr>
<tr>
<td></td>
<td>$v \leftarrow bs' + e''$</td>
</tr>
<tr>
<td></td>
<td>$\bar{v} \leftarrow \text{dbl}(v)$</td>
</tr>
<tr>
<td><strong>KEM.Decaps(s, (u, v'))</strong> :</td>
<td>$v' = \langle \bar{v} \rangle_2$</td>
</tr>
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<td>$\mu \leftarrow \text{rec}(2us, v')$</td>
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**Observe:** $2us = 2ass' + 2e's \approx 2ass' + 2es' + 2e'' \approx \bar{v}$
BCNS key exchange

- Bos, Costello, Naehrig, Stebila, IEEE S&P 2015:
  - Phrase the KEM as key exchange
  - Instantiate with concrete parameters
  - Integrate with OpenSSL $\rightarrow$ post-quantum TLS key exchange
  - Also: combined ECDH+RLWE key exchange
BCNS key exchange

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- Parameters chosen by BCNS:
  - $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n + 1)$
  - $n = 1024$
  - $q = 2^{32} - 1$
  - $\chi = D_{\mathbb{Z},\sigma}$
  - $\sigma = 8/\sqrt{2\pi} \approx 3.192$
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- Claimed security level: 128 bits pre-quantum
- Failure probability: $\approx 2^{-131072}$
A new hope

- Improve failure analysis and error reconciliation
- Choose parameters for failure probability $\approx 2^{-60}$
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- Multiple implementations
A new hope – protocol

Parameters: \( q = 12289 < 2^{14}, \) \( n = 1024 \)

Error distribution: \( \psi_{16} \)

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<td>( seed \leftarrow {0, 1}^{256} )</td>
<td>( (b, seed) \leftarrow \psi_{16}^{n} )</td>
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<td>( a \leftarrow \text{Parse}(\text{SHAKE-128}(seed)) )</td>
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<td>( k \leftarrow \text{Rec}(v', r) )</td>
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\( \mu \leftarrow \text{SHA3-256}(k) \)
Error reconciliation

- After running the protocol
  - Alice has $x_A = \text{ass}' + \text{e}'\text{s}$
  - Bob has $x_B = \text{ass}' + \text{es}' + \text{e}''$
- Those elements are similar, but not the same
- Problem: How to agree on the same key from these noisy vectors?
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- Specifically: 1 bit from 4 coefficients $\rightarrow$ 256-bit key from 1024 coefficients
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- In the following: 2-dimensional intuition (4-dim. case very similar)
- “Scale” vector $x$ to $[0, 1)^2$
2D Error reconciliation

\[(0, 0), (0, 1), (1, 1), (1, 0), \left(\frac{1}{2}, \frac{1}{2}\right)\]
2D Error reconciliation

- If $x$ is in the grey Voronoi cell: pick key bit 1
- If $x$ is in the white Voronoi cell: pick key bit 0
2D Error reconciliation

- If $x$ is in the grey Voronoi cell: pick key bit 1
- If $x$ is in the white Voronoi cell: pick key bit 0
- Reconciliation: Bob sends difference vector from $x_B$ to center of his Voronoi cell
- Alice adds this difference vector to her vector $x_A$
Discretization of reconciliation

- Sending difference vector means doubling communication
- Idea: chop Voronoi cell into $2^{dr}$ subcells
  - $d$: dimension (4 for NewHope, 2 in this picture)
  - $r$: discretization level
- Need to send only $rd$ bits per $d$ coefficients
- NewHope: $r = 2$; hence 256 bytes of reconciliation information
“Blurring the edges”

- This would all work if \( x \) was continuous uniform from \([0, 1)\)
- We start with \( x \in \{0, \ldots, q-1\}^2 \), \( q \) odd
- Odd number of possible values; no way to pick key bit without bias!
- This is the same for dimension 4
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Idea: randomly “blur the edges”

Add vector $(1/2q, 1/2q)$ with probability $1/2$ before reconciliation.

This is a generalization of Peikert’s “randomized doubling” trick.
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- Consider RLWE instance as LWE instance
- Attack using BKZ
- BKZ uses SVP oracle in smaller dimension
- Consider only the cost of one call to that oracle ("core-SVP hardness")
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- Dual attack: find short vector in dual lattice
- Length determines complexity and attacker’s advantage $\epsilon$
“I don’t like is the way that the parameters are set [. . .] I think that setting them too high impedes research.”

—anonymous reviewer
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- JarJar: instantiation with $n = 512$
- Same $q = 12289$
- Use root lattice $D_2$ instead of $D_4$
- Use $k = 24$ for the centered binomial distribution
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JarJar is not recommended for use!
# Post-quantum security

<table>
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<tr>
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<th>( m )</th>
<th>( b )</th>
<th>Known Classical</th>
<th>Known Quantum</th>
<th>Best Plausible</th>
</tr>
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<tbody>
<tr>
<td>BCNS proposal: ( q = 2^{32} - 1, n = 1024, \sigma = 3.192 )</td>
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<tr>
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<td>296</td>
<td>86</td>
<td>78</td>
<td>61</td>
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<td>281</td>
<td><strong>255</strong></td>
<td>199</td>
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</tbody>
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- \( b \): Block size for BKZ
- \( m \): Number of used samples
Against all authority

- Remember the optimization of fixed $a$?
- What if $a$ is backdoored?
- Parameter-generating authority can break key exchange
- “Solution”: Nothing-up-my-sleeves (involves endless discussion!)
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- **Must not reuse keys/noise!**
NTT-based multiplication

- Most costly arithmetic operations: multiplication in $\mathcal{R}_q$
- Idea behind selecting $n$ and $q$: fast negacyclic number-theoretic transform (NTT)
- This requires that $2n$ divides $q - 1$
- Note that $2n = 2^{11}$ divides $12288 = 2^{13} + 2^{12}$
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- NTT takes $\frac{n}{2} \log(n)$ “butterfly operations”
- Butterflies are one addition, one subtraction, one multiplication by constant
Implementation

- Very fast multiplication in $\mathcal{R}_q$: use NTT
- Define message format:
  - Send polynomials in NTT domain
  - Eliminate two of the required NTTs
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- AVX2 implementation:
  - Speed up NTT using vectorized double arithmetic
  - Use AES-256 for noise sampling
  - Use AVX2 for centered binomial
The protocol revisited

Parameters: \( q = 12289 < 2^{14} \), \( n = 1024 \)

Error distribution: \( \psi_{16}^n \)

<table>
<thead>
<tr>
<th>Alice (server)</th>
<th>Bob (client)</th>
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<tbody>
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<td>( seed \leftarrow {0, \ldots, 255}^{32} )</td>
<td>( s', e', e'' \leftarrow \psi_{16}^n )</td>
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<tr>
<td>( \hat{a} \leftarrow \text{Parse}(\text{SHAKE-128}(seed)) )</td>
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<tr>
<td>( \hat{b} \leftarrow \hat{a} \circ \hat{s} + \text{NTT}(e) )</td>
<td>( \hat{v} \leftarrow \text{NTT}^{-1}(\hat{b} \circ \hat{t}) + e'' )</td>
</tr>
<tr>
<td>( m_a = \text{encodeA}(seed, \hat{b}) \rightarrow 1824 \text{ Bytes} )</td>
<td>( (\hat{b}, seed) \leftarrow \text{decodeA}(m_a) )</td>
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<tr>
<td>( r \leftarrow \text{HelpRec}(v) )</td>
<td>( k \leftarrow \text{Rec}(v, r) )</td>
</tr>
<tr>
<td>( m_b = \text{encodeB}(\hat{u}, r) \leftarrow 2048 \text{ Bytes} )</td>
<td>( \mu \leftarrow \text{SHA3-256}(k) )</td>
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### Performance

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- Benchmarks on one core of an Intel i7-4770K (Haswell)
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- Includes around ≈ 37 000 cycles for generation of a on each side
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- Server side: $\approx 1.47\text{Mio cycles (M0)}$ and $\approx 860\text{,000 cycles (M4)}$
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Should you use NewHope?
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- Some connections from Chrome Canary to some Google services
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- See paper from PETS 2016: http://eprint.iacr.org/2015/287
- Plan: Merge these proposals
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- How about NTRU Prime?
  - Paper by Bernstein, Chuengsatiansup, Lange, van Vredendaal
  - See http://eprint.iacr.org/2016/461
  - Useful for ephemeral key exchange?
NewHope online

Paper:  https://cryptojedi.org/papers/#newhope
Software:  https://cryptojedi.org/crypto/#newhope
ARM Paper:  https://cryptojedi.org/papers/#newhopearm
ARM software:  https://github.com/newhopearm/newhopearm.git
Newhope in Go:  https://github.com/Yawning/newhope
(by Yawning Angel)
Newhope in Rust:  https://code.ciph.re/isis/newhopers
(by Isis Lovecruft)
Newhope in Java:  https://github.com/rweather/newhope-java
(by Rhys Weatherley)
Newhope in Erlang:  https://github.com/ahf/luke
(by Alexander Færøy)