Radboud University

Two approaches to verifying high-speed ECC software

Peter Schwabe
peter@cryptojedi.org
https://cryptojedi.org

April 29, 2017

"Cloudflare reported a carry bug in the P-256 implementation that they submitted for x86-64 in 7bacfc6. I can reproduce this via random testing against BoringSSL and, after applying the patch that they provided, can no longer do so, even after 2^{31} *iterations.*

This issue is not obviously exploitable, although we cannot rule out the possibility of someone managing to squeeze something through this hole. (It would be a cool paper.)"

—Adam Langley, Apr. 20, 2017

How to avoid cool papers

Peter Schwabe
peter@cryptojedi.org
https://cryptojedi.org

April 29, 2017

Carry bugs

- Brumley, Barbosa, Page, Vercauteren, 2011: exploit carry bug in OpenSSL 0.9.8g
- 633 adaptive queries to obtain entire static ECDH key

Carry bugs

- Brumley, Barbosa, Page, Vercauteren, 2011: exploit carry bug in OpenSSL 0.9.8g
- 633 adaptive queries to obtain entire static ECDH key
- CVE-2015-3193 openssl: BN_mod_exp may produce incorrect results on x86_64

Carry bugs

- Brumley, Barbosa, Page, Vercauteren, 2011: exploit carry bug in OpenSSL 0.9.8g
- 633 adaptive queries to obtain entire static ECDH key
- CVE-2015-3193 openssl: BN_mod_exp may produce incorrect results on x86_64
- CVE-2014-3570 openssl: Bignum squaring may produce incorrect results

Carry bugs

- Brumley, Barbosa, Page, Vercauteren, 2011: exploit carry bug in OpenSSL 0.9.8g
- 633 adaptive queries to obtain entire static ECDH key
- CVE-2015-3193 openssl: BN_mod_exp may produce incorrect results on x86_64
- CVE-2014-3570 openssl: Bignum squaring may produce incorrect results
- CVE-2016-7055 openssl: Carry propagating bug in Montgomery multiplication

Carry bugs

- Brumley, Barbosa, Page, Vercauteren, 2011: exploit carry bug in OpenSSL 0.9.8g
- 633 adaptive queries to obtain entire static ECDH key
- CVE-2015-3193 openssl: BN_mod_exp may produce incorrect results on x86_64
- CVE-2014-3570 openssl: Bignum squaring may produce incorrect results
- CVE-2016-7055 openssl: Carry propagating bug in Montgomery multiplication
- CVE-2017-3732 openssl: BN_mod_exp may produce incorrect results on x86_64

Carry bugs

- Brumley, Barbosa, Page, Vercauteren, 2011: exploit carry bug in OpenSSL 0.9.8g
- 633 adaptive queries to obtain entire static ECDH key
- CVE-2015-3193 openssl: BN_mod_exp may produce incorrect results on x86_64
- CVE-2014-3570 openssl: Bignum squaring may produce incorrect results
- CVE-2016-7055 openssl: Carry propagating bug in Montgomery multiplication
- CVE-2017-3732 openssl: BN_mod_exp may produce incorrect results on x86_64
- Ed25519 and X25519 "amd64-64" implementation (CHES 2011)

Example for today: X25519

- Bernstein 2006: X25519 Diffie-Hellman key exchange (originally: "Curve25519")
- Secret keys: 32-byte little-endian scalars
- Public keys: 32-byte arrays, encoding x-coordinate of a point on

$$E : y^2 = x^3 + 486662x^2 + x$$

  over $\mathbb{F}_{2^{255}-19}$
- Base point: $(9, 0, \ldots, 0)$

Example for today: X25519

- Bernstein 2006: X25519 Diffie-Hellman key exchange (originally: "Curve25519")
- Secret keys: 32-byte little-endian scalars
- Public keys: 32-byte arrays, encoding x-coordinate of a point on

$$E : y^2 = x^3 + 486662x^2 + x$$

  over $\mathbb{F}_{2^{255}-19}$
- Base point: $(9, 0, \ldots, 0)$
- Given secret key s and public key (or base point) P:
  - Copy s to s'
  - Set least significant 3 bits of s' to zero
  - Set most significant bit of s' to zero
  - Set second-most significant bit of s' to one
  - Compute x-coordinate of $s'P$

Require: A scalar $0 \leq k \in \mathbb{Z}$ and the x-coordinate x_P of some point P

Ensure: x_{kP}

$X_1 = x_P$; $X_2 = 1$; $Z_2 = 0$; $X_3 = x_P$; $Z_3 = 1$

for $i \leftarrow n - 1$ downto 0 **do**

  if bit i of k is 1 **then**

    $(X3, Z3, X2, Z2) \leftarrow \text{ladderstep}(X1, X3, Z3, X2, Z2)$

  else

    $(X2, Z2, X3, Z3) \leftarrow \text{ladderstep}(X1, X2, Z2, X3, Z3)$

  end if

end for

return $X_2 \cdot Z_2^{-1}$

One Montgomery "ladder step"

const $a24 = (A+2)/4$ (A from the curve equation)
function ladderstep($X_{Q-P}, X_P, Z_P, X_Q, Z_Q$)

$t_1 \leftarrow X_P + Z_P$
$t_6 \leftarrow t_1^2$
$t_2 \leftarrow X_P - Z_P$
$t_7 \leftarrow t_2^2$
$t_5 \leftarrow t_6 - t_7$
$t_3 \leftarrow X_Q + Z_Q$
$t_4 \leftarrow X_Q - Z_Q$
$t_8 \leftarrow t_4 \cdot t_1$
$t_9 \leftarrow t_3 \cdot t_2$
$X_{P+Q} \leftarrow (t_8 + t_9)^2$
$Z_{P+Q} \leftarrow X_{Q-P} \cdot (t_8 - t_9)^2$
$X_{2P} \leftarrow t_6 \cdot t_7$
$Z_{2P} \leftarrow t_5 \cdot (t_7 + a24 \cdot t_5)$
return $(X_{2P}, Z_{2P}, X_{P+Q}, Z_{P+Q})$

end function

Curve25519 implementations

- Bernstein, 2006: X25519 for various 32-bit Intel and AMD processors
- Gaudry, Thomé, 2007: X25519 for 64-bit Intel and AMD processors
- Costigan, Schwabe, 2009: X25519 for Cell Broadband Engine
- Bernstein, Duif, Lange, Schwabe, Yang, 2011: X25519 for Intel Nehalem/Westmere
- Düll, Haase, Hinterwälder, Hutter, Paar, Sánchez, Schwabe, 2015: X25519 for AVR ATmega, TI MSP430, and ARM Cortex-M0
- Chou, 2015: The fastest Curve25519 software ever
- Many more implementations, most without scientific papers

Curve25519 implementations

- Bernstein, 2006: X25519 for various 32-bit Intel and AMD processors
- Gaudry, Thomé, 2007: X25519 for 64-bit Intel and AMD processors
- Costigan, Schwabe, 2009: X25519 for Cell Broadband Engine
- Bernstein, Duif, Lange, Schwabe, Yang, 2011: X25519 for Intel Nehalem/Westmere
- Düll, Haase, Hinterwälder, Hutter, Paar, Sánchez, Schwabe, 2015: X25519 for AVR ATmega, TI MSP430, and ARM Cortex-M0
- Chou, 2015: The fastest Curve25519 software ever
- Many more implementations, most without scientific papers

Arithmetic in $\mathbb{F}_{2^{255}-19}$ for AMD64

Radix 2^{64}

- Standard: break elements of $\mathbb{F}_{2^{255}-19}$ into 4 64-bit integers
- (Schoolbook) multiplication breaks down into 16 64-bit integer multiplications
- Adding up partial results requires many add-with-carry (`adc`)
- Westmere bottleneck: 1 `adc` every two cycles vs. 3 `add` per cycle

Arithmetic in $\mathbb{F}_{2^{255}-19}$ for AMD64

Radix 2^{64}

- Standard: break elements of $\mathbb{F}_{2^{255}-19}$ into 4 64-bit integers
- (Schoolbook) multiplication breaks down into 16 64-bit integer multiplications
- Adding up partial results requires many add-with-carry (`adc`)
- Westmere bottleneck: 1 `adc` every two cycles vs. 3 `add` per cycle

Radix 2^{51}

- Instead, break into 5 64-bit integers, use radix 2^{51}
- Can delay carry operations; carry "en bloc"
- Schoolbook multiplication now 25 64-bit integer multiplications
- Easy to merge multiplication with reduction (multiplies by 19)
- Better performance on Westmere/Nehalem, worse on 65 nm Core 2 and AMD processors

Bug in the radix-64 reduction

```
mulq  crypto_sign_ed25519_amd64_64_38
add  %rax,%r13
adc %rdx,%r14
adc $0,%r14
mov  %r9,%rax
mulq  crypto_sign_ed25519_amd64_64_38
add  %rax,%r14
adc %rdx,%r15
adc $0,%r15
mov  %r10,%rax
mulq  crypto_sign_ed25519_amd64_64_38
add  %rax,%r15
adc %rdx,%rbx
adc $0,%rbx
mov  %r11,%rax
mulq  crypto_sign_ed25519_amd64_64_38
add  %rax,%rbx
mov  $0,%rsi
adc %rdx,%rsi
```

Bug in the radix-64 reduction

```
(uint128) mulrdx mulrax = mulrax * *(uint64 *)&crypto_sign_ed25519_amd64_64_38
carry? r0 += mulrax
carry? r1 += mulrdx + carry
r1 += 0 + carry
mulrax = mulr5
(uint128) mulrdx mulrax = mulrax * *(uint64 *)&crypto_sign_ed25519_amd64_64_38
carry? r1 += mulrax
carry? r2 += mulrdx + carry
r2 += 0 + carry
mulrax = mulr6
(uint128) mulrdx mulrax = mulrax * *(uint64 *)&crypto_sign_ed25519_amd64_64_38
carry? r2 += mulrax
carry? r3 += mulrdx + carry
r3 += 0 + carry
mulrax = mulr7
(uint128) mulrdx mulrax = mulrax * *(uint64 *)&crypto_sign_ed25519_amd64_64_38
carry? r3 += mulrax
mulr4 = 0
mulr4 += mulrdx + carry
```

Bug in the radix-64 reduction

```
(uint128) mulrdx mulrax = mulrax * *(uint64 *)&crypto_sign_ed25519_amd64_64_38
carry? r0 += mulrax
carry? r1 += mulrdx + carry
r1 += 0 + carry
mulrax = mulr5
(uint128) mulrdx mulrax = mulrax * *(uint64 *)&crypto_sign_ed25519_amd64_64_38
carry? r1 += mulrax
carry? r2 += mulrdx + carry
r2 += 0 + carry
mulrax = mulr6
(uint128) mulrdx mulrax = mulrax * *(uint64 *)&crypto_sign_ed25519_amd64_64_38
carry? r2 += mulrax
carry? r3 += mulrdx + carry
r3 += 0 + carry
mulrax = mulr7
(uint128) mulrdx mulrax = mulrax * *(uint64 *)&crypto_sign_ed25519_amd64_64_38
carry? r3 += mulrax
mulr4 = 0
mulr4 += mulrdx + carry
```

Full software package contains 8912 lines of qhasm code!

Testing

- Is cheap, catches many bugs
- Does not conflict with performance
- Provides very high confidence in correctness for *some* crypto algorithms
- Typically fails to catch very rarely triggered bugs

Audits

- Expensive (time and/or money)
- Conflicts with performance
- Standard approach to ensure correctness and quality of crypto software

Formal verification

- Strongest guarantees of correctness
- Probably conflicts with performance

Directions to correct crypto

Formal verification

- Strongest guarantees of correctness
- Probably conflicts with performance
- **Should focus on cases where tests fail**

Verification: the vision

- C or assembly programmer adds high-level annotations
- More specifically, for example:
  - Limbs $a_0, \ldots, a_n$ compose a field element A
  - Limbs $b_0, \ldots, b_n$ compose a field element B
  - Limbs $r_0, \ldots, r_n$ compose a field element R
  - $R = A \cdot B$

Verification: the vision

- C or assembly programmer adds high-level annotations
- More specifically, for example:
    - Limbs $a_0, \ldots, a_n$ compose a field element A
    - Limbs $b_0, \ldots, b_n$ compose a field element B
    - Limbs $r_0, \ldots, r_n$ compose a field element R
    - $R = A \cdot B$
- Annotated code gets fed to verification tool
- Verification ensures that operation on limbs corresponds to high-level arithmetic
- Audits look at high-level annotations

Verification: the vision

- C or assembly programmer adds high-level annotations
- More specifically, for example:
  - Limbs $a_0, \ldots, a_n$ compose a field element A
  - Limbs $b_0, \ldots, b_n$ compose a field element B
  - Limbs $r_0, \ldots, r_n$ compose a field element R
  - $R = A \cdot B$
- Annotated code gets fed to verification tool
- Verification ensures that operation on limbs corresponds to high-level arithmetic
- Audits look at high-level annotations
- Even better: feed to even higher level verification
- Verify that the sequence of field operations accomplishes EC arithmetic

Verification approach I

Joint work with Yu-Fang Chen, Chang-Hong Hsu, Hsin-Hung Lin, Ming-Hsien Tsai, Bow-Yaw Wang, Bo-Yin Yang, and Shang-Yi Yang.

Verification approach I

Joint work with Yu-Fang Chen, Chang-Hong Hsu, Hsin-Hung Lin, Ming-Hsien Tsai, Bow-Yaw Wang, Bo-Yin Yang, and Shang-Yi Yang.

- Nehalem Curve25519 software is written in `qhasm`
- `qhasm` is a portable assembly language by Bernstein

Verification approach I

Joint work with Yu-Fang Chen, Chang-Hong Hsu, Hsin-Hung Lin, Ming-Hsien Tsai, Bow-Yaw Wang, Bo-Yin Yang, and Shang-Yi Yang.

- Nehalem Curve25519 software is written in `qhasm`
- `qhasm` is a portable assembly language by Bernstein
- Idea for verification:
  - Annotate `qhasm` code
  - Translate annotated `qhasm` automatically to SMT-solver `boolector`
  - Use `boolector` to verify software

Verification approach I

Joint work with Yu-Fang Chen, Chang-Hong Hsu, Hsin-Hung Lin, Ming-Hsien Tsai, Bow-Yaw Wang, Bo-Yin Yang, and Shang-Yi Yang.

- Nehalem Curve25519 software is written in `qhasm`
- `qhasm` is a portable assembly language by Bernstein
- Idea for verification:
  - Annotate `qhasm` code
  - Translate annotated `qhasm` automatically to SMT-solver `boolector`
  - Use `boolector` to verify software
- Verification target: Montgomery ladder step of X25519:
  - 5 multiplications in $\mathbb{F}_{2^{255}-19}$
  - 4 squarings in $\mathbb{F}_{2^{255}-19}$
  - 1 multiplication by 121666
  - Several additions and subtractions

Example: Addition in radix 2^{51}

```
#// assume 0 <=u x0, x1, x2, x3, x4 <=u 2**51 + 2**15
#// assume 0 <=u y0, y1, y2, y3, y4 <=u 2**51 + 2**15
r0 = x0
r1 = x1
r2 = x2
r3 = x3
r4 = x4
r0 += y0
r1 += y1
r2 += y2
r3 += y3
r4 += y4
#// var sum_x = x0@u320 + x1@u320 * 2**51 + x2@u320 * 2**102 \
                + x3@u320 * 2**153 + x4@u320 * 2**204
#//     sum_y = y0@u320 + y1@u320 * 2**51 + y2@u320 * 2**102 \
                + y3@u320 * 2**153 + y4@u320 * 2**204
#//     sum_r = r0@u320 + r1@u320 * 2**51 + r2@u320 * 2**102 \
                + r3@u320 * 2**153 + r4@u320 * 2**204
#// assert (sum_r - (sum_x + sum_y)) % (2**255 - 19) = 0 &&
#//        0 <=u r0, r1, r2, r3, r4 <u 2**53
```

How about multiplication?

- Again, express input field elements and output field elements
- Again, express bounds on the "limb size"
- Again, express algebraic relation of a modular multiplication
- Overall slightly more annoations for an auditor to look at

How about multiplication?

- Again, express input field elements and output field elements
- Again, express bounds on the “limb size”
- Again, express algebraic relation of a modular multiplication
- Overall slightly more annoations for an auditor to look at
- *Huge amount* of intermediate annotations
- SMT solver cannot simply verify from inputs to outputs

How about multiplication?

- Again, express input field elements and output field elements
- Again, express bounds on the “limb size”
- Again, express algebraic relation of a modular multiplication
- Overall slightly more annoations for an auditor to look at
- *Huge amount* of intermediate annotations
- SMT solver cannot simply verify from inputs to outputs
- Overall:
  - 217 lines of `qhasm`, including variable declarations
  - 589 lines of annotations

How about multiplication?

- Again, express input field elements and output field elements
- Again, express bounds on the "limb size"
- Again, express algebraic relation of a modular multiplication
- Overall slightly more annoations for an auditor to look at
- *Huge amount* of intermediate annotations
- SMT solver cannot simply verify from inputs to outputs
- Overall:
  - 217 lines of `qhasm`, including variable declarations
  - 589 lines of annotations
- Large amount of manual work on top of assembly optimization
- Writing verifiable code requires expert knowledge from two domains!
- Verification of just multiplication takes > 90 hours

Overall results

- Formally verified Montgomery ladderstep
  - "Redundant" radix-2^{51} representation
  - Non-redundant radix-2^{64} representation
  - Reproduced bug in original version of the software
- Most verification used automatic `qhasm` $\rightarrow$ `boolector` translation
- Tiny bit of code in radix-2^{64} needed proof assistant Coq

Verification approach II

- 2 problems with SMT approach:
  - Huge amount of (manual) annotations
  - Long verification time

Verification approach II

- 2 problems with SMT approach:
  - Huge amount of (manual) annotations
  - Long verification time
- Idea: automagically translate to input for computer-algebra system
- Accept failures to prove correctness

Verification approach II

- 2 problems with SMT approach:
  - Huge amount of (manual) annotations
  - Long verification time
- Idea: automagically translate to input for computer-algebra system
- Accept failures to prove correctness

Work in progress with Bernstein

- Annotate C code (later: also `qhasm`)
- (Currently) use C++ compiler and operator overloading to
  - Keep track of computation graph
  - Keep track of worst-case ranges of limbs
  - Output polynomial relations to Sage
  - Use Sage to verify correctness of C code

Example: addition (radix $2^{25.5}$)

```
crypto_int32 f[10];
crypto_int32 g[10];
crypto_int32 h[10];

verifier_bigint vf;
verifier_addlimbs_10_255(&vf,f);
verifier_bigint vg;
verifier_addlimbs_10_255(&vg,g);

fe_add(h,f,g);

verifier_bigint vh;
verifier_addlimbs_10_255(&vh,h);
verifier_assertsum(&vh,&vf,&vg);
```

Example: multiplication

```
crypto_int32 f[10];
crypto_int32 g[10];
crypto_int32 h[10];

verifier_bigint vf;
verifier_addlimbs_10_255(&vf,f);
verifier_bigint vg;
verifier_addlimbs_10_255(&vg,g);

fe_mul(h,f,g);

verifier_bigint vh;
verifier_addlimbs_10_255(&vh,h);
verifier_assertprodmod(&vh,&vf,&vg,"2^255-19");
```

A small demo

- Consider computation of $x^{2^{100}}$ in $\mathbb{F}_{2^{127}-1}$
- Input is little-endian byte array
- Convert to internal representation in radix 2^{26}

A small demo

- Consider computation of $x^{2^{100}}$ in $\mathbb{F}_{2^{127}-1}$
- Input is little-endian byte array
- Convert to internal representation in radix 2^{26}
- Verify a single squaring

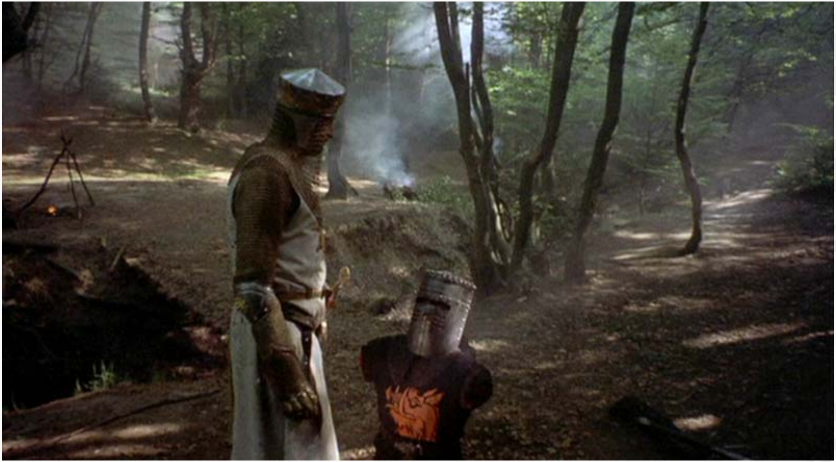
A small demo

- Consider computation of $x^{2^{100}}$ in $\mathbb{F}_{2^{127}-1}$
- Input is little-endian byte array
- Convert to internal representation in radix 2^{26}
- Verify a single squaring
- Put a loop around it

A small demo

- Consider computation of $x^{2^{100}}$ in $\mathbb{F}_{2^{127}-1}$
- Input is little-endian byte array
- Convert to internal representation in radix 2^{26}
- Verify a single squaring
- Put a loop around it
- Still too slow for big chunks of code
  - Problem: verification always goes back to the beginning
  - Idea: Declare that we trust already verified results
  - This is known as "cutting" the verification

Let's call it a draw



Back to X25519

```
fe_sub(tmp0,x3,z3);

  verifier_bigint D;
  verifier_limbs_10_255(&D,tmp0);
  verifier_assertdiff(&D,&X3,&Z3);
  verifier_cutlimbs(tmp0,10);

fe_sub(tmp1,x2,z2);

  verifier_bigint B;
  verifier_limbs_10_255(&B,tmp1);
  verifier_assertdiff(&B,&X2,&Z2);
  verifier_cutlimbs(tmp1,10);

fe_add(x2,x2,z2);

  verifier_bigint A;
  verifier_limbs_10_255(&A,x2);
  verifier_assertsum(&A,&X2,&Z2);
  verifier_cutlimbs(x2,10);

fe_add(z2,x3,z3);

  verifier_bigint C;
  verifier_limbs_10_255(&C,z2);
  verifier_assertsum(&C,&X3,&Z3);
  verifier_cutlimbs(z2,10);

fe_mul(z3,tmp0,x2);

  verifier_bigint DA;
  verifier_limbs_10_255(&DA,z3);
  verifier_assertprodmod(&DA,&D,&A,vp);
  verifier_cutlimbs(z3,10);

fe_mul(z2,z2,tmp1);

  verifier_bigint CB;
  verifier_limbs_10_255(&CB,z2);
  verifier_assertprodmod(&CB,&C,&B,vp);
  verifier_cutlimbs(z2,10);
```

- Input conversion from byte array (see $\mathbb{F}_{2^{127}-1}$ example)

Beyond the ladder step

- Input conversion from byte array (see $\mathbb{F}_{2^{127}-1}$ example)
- "Clamping" of scalar: currently not covered

Beyond the ladder step

- Input conversion from byte array (see $\mathbb{F}_{2^{127}-1}$ example)
- "Clamping" of scalar: currently not covered
- Final inversion: exponentiation by $p - 2$

Beyond the ladder step

- Input conversion from byte array (see $\mathbb{F}_{2^{127}-1}$ example)
- "Clamping" of scalar: currently not covered
- Final inversion: exponentiation by $p - 2$
- "Freezing" of elements:
  - Carry to produce result in $\{0, \ldots, 2^{255} - 1\}$
  - Conditionally subtract p
  - Use `fork` to verify both cases

High-level verification

```
# feed through: ./unroll x1 n
p = 2**255-19
A = 486662
x2,z2,x3,z3 = 1,0,x1,1
for i in reversed(range(255)):
  ni = bit(n,i)
  x2,x3 = cswap(x2,x3,ni)
  z2,z3 = cswap(z2,z3,ni)
  x3,z3 = (4*(x2*x3-z2*z3)**2,4*x1*(x2*z3-z2*x3)**2)
  x2,z2 = ((x2**2-z2**2)**2,4*x2*z2*(x2**2+A*x2*z2+z2**2))
  x3,z3 = (x3%p,z3%p)
  x2,z2 = (x2%p,z2%p)
  cut(x2)
  cut(x3)
  cut(z2)
  cut(z3)
  x2,x3 = cswap(x2,x3,ni)
  z2,z3 = cswap(z2,z3,ni)
cut(x2)
cut(z2)
return x2*pow(z2,p-2,p)
```

First results and TODOs

Results

- Verification of modular multiplication in a few seconds
- Verification of full X25519 Montgomery ladder in ≈1:10 minutes
- Verification against high-level code

First results and TODOs

Results

- Verification of modular multiplication in a few seconds
- Verification of full X25519 Montgomery ladder in ≈1:10 minutes
- Verification against high-level code

TODOs

- Support assembly/`qhasm`
- Get rid of C++ compiler
- Support "non-redundant" arithmetic
- Support window methods
- Test, test, test

Papers and Software

- Yu-Fang Chen, Chang-Hong Hsu, Hsin-Hung Lin, Peter Schwabe, Ming-Hsien Tsai, Bow-Yaw Wang, Bo-Yin Yang, and Shang-Yi Yang. *Verifying Curve25519 Software*.
  https://cryptojedi.org/papers/#verify25519
- Many X25519 implementations in SUPERCOP (`crypto_scalarmult/curve25519`)
  https://bench.cr.yp.to/supercop.html
- Verification using `boolector`:
  https://cryptojedi.org/crypto/#verify25519
- Verification with `gfverif`:
  https://cryptojedi.org/crypto/#gfverif