An Introduction to hash-based signatures

Peter Schwabe
December 7, 2021
So many NIST candidates and one thing they all have in common...
So many NIST candidates and one thing they all have in common... they all need a hash function.
So many NIST candidates and one thing they all have in common... they all need a hash function.

What can we do with just a hash function?
Hash-based signatures

- Hash functions map long strings to fixed-length strings
- Standard properties required from a cryptographic hash function:
  - **Collision resistance**: Hard two find two inputs that produce the same output
  - **Preimage resistance**: Given the output, it’s hard to find the input
  - **2nd preimage resistance**: Given input and output, it’s hard to find a second input, producing the same output
• Hash functions map long strings to fixed-length strings

• Standard properties required from a cryptographic hash function:
  • **Collision resistance**: Hard to find two inputs that produce the same output
  • **Preimage resistance**: Given the output, it’s hard to find the input
  • **2nd preimage resistance**: Given input and output, it’s hard to find a second input, producing the same output

• Collision resistance is stronger assumption than (2nd) preimage resistance

• Ideally, don’t want to rely on collision resistance
Signatures for 0-bit messages

Key generation

- Generate 256-bit random value \( r \) (secret key)
- Compute \( p = h(r) \) (public key)
Signatures for 0-bit messages

Key generation

- Generate 256-bit random value \( r \) (secret key)
- Compute \( p = h(r) \) (public key)

Signing

- Send \( \sigma = r \)
Signatures for 0-bit messages

Key generation
- Generate 256-bit random value $r$ (secret key)
- Compute $p = h(r)$ (public key)

Signing
- Send $\sigma = r$

Verification
- Check that $h(r) = p$
Security of this scheme

• Clearly an attacker who can invert \( h \) can break the scheme
• Can we reduce from preimage-resistance to unforgeability?
Security of this scheme

- Clearly an attacker who can invert $h$ can break the scheme
- Can we reduce from preimage-resistance to unforgeability?
- Proof game:
  - Assume oracle $\mathcal{A}$ that computes forgery, given public key $pk$
  - Get input $y$, use oracle to compute $x$, s.t., $h(x) = y$
  - Idea: use public-key $pk = y$, oracle will compute forgery $x$
Security of this scheme

• Clearly an attacker who can invert $h$ can break the scheme
• Can we reduce from preimage-resistance to unforgeability?
• Proof game:
  • Assume oracle $\mathcal{A}$ that computes forgery, given public key $pk$
  • Get input $y$, use oracle to compute $x$, s.t., $h(x) = y$
  • Idea: use public-key $pk = y$, oracle will compute forgery $x$
  • . . . or will it?
Security of this scheme

• Clearly an attacker who can invert $h$ can break the scheme
• Can we reduce from preimage-resistance to unforgeability?
• Proof game:
  • Assume oracle $\mathcal{A}$ that computes forgery, given public key $pk$
  • Get input $y$, use oracle to compute $x$, s.t., $h(x) = y$
  • Idea: use public-key $pk = y$, oracle will compute forgery $x$
  • . . . or will it?
• Problem: $y$ is not an output of $h$
• What if $\mathcal{A}$ can distinguish legit $pk$ from random?
• Need additional property of $h$: undetectability
• From now on assume that all our hash functions are undetectable
Signatures for 1-bit messages

Key generation

- Generate 256-bit random values \((r_0, r_1) = s\) (secret key)
- Compute \((h(r_0), h(r_1)) = (p_0, p_1) = p\) (public key)
Signatures for 1-bit messages

Key generation

• Generate 256-bit random values \((r_0, r_1) = s\) (secret key)
• Compute \((h(r_0), h(r_1)) = (p_0, p_1) = p\) (public key)

Signing

• Signature for message \(b = 0\): \(\sigma = r_0\)
• Signature for message \(b = 1\): \(\sigma = r_1\)
Signatures for 1-bit messages

Key generation

- Generate 256-bit random values \((r_0, r_1) = s\) (secret key)
- Compute \((h(r_0), h(r_1)) = (p_0, p_1) = p\) (public key)

Signing

- Signature for message \(b = 0\): \(\sigma = r_0\)
- Signature for message \(b = 1\): \(\sigma = r_1\)

Verification
Check that \(h(\sigma) = p_b\)
Security of this scheme

• Same idea as for 0-bit messages: reduce from preimage resistance
Security of this scheme

• Same idea as for 0-bit messages: reduce from preimage resistance
• Proof game:
  • Assume oracle $\mathcal{A}$ that computes forgery, given public key $pk$
  • Get input $y$, use “public key” $(h(r_0), y)$ or $(y, h(r_1))$
Security of this scheme

• Same idea as for 0-bit messages: reduce from preimage resistance
• Proof game:
  • Assume oracle $\mathcal{A}$ that computes forgery, given public key $pk$
  • Get input $y$, use “public key” $(h(r_0), y)$ or $(y, h(r_1))$
  • $\mathcal{A}$ asks for signature on either 0 or 1
  • If you can, answer with preimage, otherwise fail (abort)
• Same idea as for 0-bit messages: reduce from preimage resistance

• Proof game:
  • Assume oracle $\mathcal{A}$ that computes forgery, given public key $pk$
  • Get input $y$, use “public key” $(h(r_0), y)$ or $(y, h(r_1))$
  • $\mathcal{A}$ asks for signature on either 0 or 1
  • If you can, answer with preimage, otherwise fail (abort)
  • Now $\mathcal{A}$ returns preimage, i.e., preimage of $y$
Security of this scheme

• Same idea as for 0-bit messages: reduce from preimage resistance

• Proof game:
  • Assume oracle $\mathcal{A}$ that computes forgery, given public key $pk$
  • Get input $y$, use “public key” $(h(r_0), y)$ or $(y, h(r_1))$
  • $\mathcal{A}$ asks for signature on either 0 or 1
  • If you can, answer with preimage, otherwise fail (abort)
  • Now $\mathcal{A}$ returns preimage, i.e., preimage of $y$

• Reduction only works with $1/2$ probability

• We get a **tightness loss** of $1/2$
One-time signatures for 256-bit messages

Key generation

- Generate 256-bit random values $s = (r_{0,0}, r_{0,1}, \ldots, r_{255,0}, r_{255,1})$
- Compute $p = (h(r_{0,0}), h(r_{0,1}), \ldots, h(r_{255,0}), h(r_{255,1})) = (p_{0,0}, p_{0,1}, \ldots, p_{255,0}, p_{255,1})$
One-time signatures for 256-bit messages

Key generation

- Generate 256-bit random values \( s = (r_{0,0}, r_{0,1}, \ldots, r_{255,0}, r_{255,1}) \)
- Compute \( p = (h(r_{0,0}), h(r_{0,1}), \ldots, h(r_{255,0}), h(r_{255,1})) = (p_{0,0}, p_{0,1}, \ldots, p_{255,0}, p_{255,1}) \)

Signing

- Signature for message \((b_0, \ldots, b_{255})\):
  \[ \sigma = (\sigma_0, \ldots, \sigma_{255}) = (r_{0,b_0}, \ldots, r_{255,b_{255}}) \]
One-time signatures for 256-bit messages

Key generation

- Generate 256-bit random values $s = (r_{0,0}, r_{0,1}, \ldots, r_{255,0}, r_{255,1})$
- Compute $p = (h(r_{0,0}), h(r_{0,1}), \ldots, h(r_{255,0}), h(r_{255,1})) = (p_{0,0}, p_{0,1}, \ldots, p_{255,0}, p_{255,1})$

Signing

- Signature for message $(b_0, \ldots, b_{255})$:
  $\sigma = (\sigma_0, \ldots, \sigma_{255}) = (r_{0,b_0}, \ldots, r_{255,b_{255}})$

Verification

- Check that $h(\sigma_0) = p_{0,b_0}$
- \ldots
- Check that $h(\sigma_{255}) = p_{255,b_{255}}$
Security of this scheme

- Same idea as before, replace one $p_{j,b}$ in the public key by challenge $y$
- Fail if signing needs the preimage of $y$
- In forgery, attacker has to flip at least one bit in $m$
- Chance of $1/256$ that attacker flips the bit with the challenge
- Overall tightness loss of $1/512$
Winternitz OTS (basic idea)

- Lamport signatures are rather large (8 KB)
- Can we tradeoff speed for size?
- Idea: use $h^w(r)$ instead of $h(r)$ (“hash chains”)
Winternitz OTS (basic idea)

- Lamport signatures are rather large (8 KB)
- Can we tradeoff speed for size?
- Idea: use $h^w(r)$ instead of $h(r)$ ("hash chains")

**Key generation**

- Generate 256-bit random values $r_0, \ldots, r_{63}$ (secret key)
- Compute $(p_0, \ldots, p_{63}) = (h^{15}(r_0), \ldots, h^{15}(r_{63})$ (public key)
Winternitz OTS (basic idea)

- Lamport signatures are rather large (8 KB)
- Can we tradeoff speed for size?
- Idea: use $h^w(r)$ instead of $h(r)$ (“hash chains”)

Key generation

- Generate 256-bit random values $r_0, \ldots, r_{63}$ (secret key)
- Compute $(p_0, \ldots, p_{63}) = (h^{15}(r_0), \ldots, h^{15}(r_{63})$ (public key)

Signing

- Chop 256 bit message into 64 chunks of 4 bits $m = (m_0, \ldots, m_{63})$
- Compute $\sigma = (\sigma_0, \ldots, \sigma_{63}) = (h^{m_0}(r_0), \ldots, h^{m_{63}}(r_{63}))$
Winternitz OTS (basic idea)

• Lamport signatures are rather large (8 KB)
• Can we tradeoff speed for size?
• Idea: use $h^w(r)$ instead of $h(r)$ (“hash chains”)

Key generation

• Generate 256-bit random values $r_0, \ldots, r_{63}$ (secret key)
• Compute $(p_0, \ldots, p_{63}) = (h^{15}(r_0), \ldots, h^{15}(r_{63}))$ (public key)

Signing

• Chop 256-bit message into 64 chunks of 4 bits $m = (m_0, \ldots, m_{63})$
• Compute $\sigma = (\sigma_0, \ldots, \sigma_{63}) = (h^{m_0}(r_0), \ldots, h^{m_{63}}(r_{63}))$

Verification

• Check that $p_0 = h^{15-m_0}(\sigma_0), \ldots, p_{63} = h^{15-m_{63}}(\sigma_{63})$
Winternitz OTS (basic idea, ctd.)

\[ h^{15}(r_0) \]
\[ h^{14}(r_0) \]
\[ h(r_0) \]
\[ r_0 \]

\[ h^{15}(r_1) \]
\[ h^{14}(r_1) \]
\[ h(r_1) \]
\[ r_1 \]

\[ h^{15}(r_{63}) \]
\[ h^{14}(r_{63}) \]
\[ h(r_{63}) \]
\[ r_{63} \]
Winternitz OTS (making it secure)

- Once you signed, say, $m = (8, m_1, \ldots, m_{63})$, can easily forge signature on $m = (9, m_1, \ldots, m_{63})$
- Idea: introduce checksum, force attacker to “go down” some chain in exchange
Winternitz OTS (making it secure)

• Once you signed, say, $m = (8, m_1, \ldots, m_{63})$, can easily forge signature on $m = (9, m_1, \ldots, m_{63})$

• Idea: introduce checksum, force attacker to “go down” some chain in exchange

• Compute $c = 960 - \sum_{i=0}^{63} m_i \in \{0, \ldots, 960\}$

• Write $c$ in radix 16, obtain $c_0, c_1, c_2$

• Compute hash chains for $c_0, c_1, c_2$ as well
• Once you signed, say, $m = (8, m_1, \ldots, m_{63})$, can easily forge signature on $m = (9, m_1, \ldots, m_{63})$

• Idea: introduce checksum, force attacker to “go down” some chain in exchange

• Compute $c = 960 - \sum_{i=0}^{63} m_i \in \{0, \ldots, 960\}$

• Write $c$ in radix 16, obtain $c_0, c_1, c_2$

• Compute hash chains for $c_0, c_1, c_2$ as well

• When increasing one of the $m_i$’s, one of the $c_i$’s decreases

• In total obtain 67 hash chains, signatures have 2144 bytes
• The value $w = 16$ (15 hashes per chain) is tunable
• Can also use, e.g., 256 (chop message into bytes)
WOTS notes

- The value $w = 16$ (15 hashes per chain) is tunable
- Can also use, e.g., 256 (chop message into bytes)
- Lots of tradeoffs between speed and size
  - $w = 16$ yields $\approx 2.1$ KB signatures
  - $w = 256$ yields $\approx 1.1$ KB signatures
  - However, $w = 256$ makes signing and verification $\approx 8 \times$ slower
• The value $w = 16$ (15 hashes per chain) is tunable
• Can also use, e.g., 256 (chop message into bytes)
• Lots of tradeoffs between speed and size
  • $w = 16$ yields $\approx 2.1$ KB signatures
  • $w = 256$ yields $\approx 1.1$ KB signatures
  • However, $w = 256$ makes signing and verification $\approx 8 \times$ slower
• Verification recovers (and compares) the full public key
• Can publish $h(pk)$ instead of $pk$
From WOTS to WOTS⁺

- An attacker who can compute preimages can go backwards in chains
- Problem: hard to prove that this is the only way to forge
From WOTS to WOTS+

• An attacker who can compute preimages can go backwards in chains
• Problem: hard to prove that this is the only way to forge
• Proof needs to go the other way round
• Given forgery oracle, need to compute preimage for some given \( x \)
• Can again place preimage challenge anywhere inside the chains
From WOTS to WOTS⁺

- An attacker who can compute preimages can go backwards in chains
- Problem: hard to prove that this is the only way to forge
- Proof needs to go the other way round
- Given forgery oracle, need to compute preimage for some given $x$
- Can again place preimage challenge anywhere inside the chains
- Problem: two ways for oracle to forge:
  - compute preimage (solve challenge)
  - find different chain that collides further up
- Forgery gives us either preimage or collision

Idea (Hülsing, 2013): control one input in that collision, get 2nd preimage!

Replace $h(r)$ by $h(r \oplus b)$ for “bitmask” $b$

Include bitmasks in public key

Reduction can now choose inputs to hash function
From WOTS to WOTS+

- An attacker who can compute preimages can go backwards in chains
- Problem: hard to prove that this is the only way to forge
- Proof needs to go the other way round
- Given forgery oracle, need to compute preimage for some given x
- Can again place preimage challenge anywhere inside the chains
- Problem: two ways for oracle to forge:
  - compute preimage (solve challenge)
  - find different chain that collides further up
- Forgery gives us either preimage or collision
- Idea (Hülsing, 2013): control one input in that collision, get 2nd preimage!
From WOTS to WOTS+

- An attacker who can compute preimages can go backwards in chains
- Problem: hard to prove that this is the only way to forge
- Proof needs to go the other way round
- Given forgery oracle, need to compute preimage for some given $x$
- Can again place preimage challenge anywhere inside the chains
- Problem: two ways for oracle to forge:
  - compute preimage (solve challenge)
  - find different chain that collides further up
- Forgery gives us either preimage or collision
- Idea (Hülsing, 2013): control one input in that collision, get 2nd preimage!
- Replace $h(r)$ by $h(r \oplus b)$ for “bitmask” $b$
- Include bitmasks in public key
- Reduction can now choose inputs to hash function
How about the message hash?

• What if we want to sign messages longer than 256 bits?
• Simple answer: sign $h(m)$
• Requires collision-resistant hash-function $h$
How about the message hash?

• What if we want to sign messages longer than 256 bits?
• Simple answer: sign $h(m)$
• Requires collision-resistant hash-function $h$
• Idea: randomize before feeding $m$ into $h$
  • Pick random $r$
  • Compute $h(r \mid m)$
  • Send $r$ as part of the signature
How about the message hash?

• What if we want to sign messages longer than 256 bits?
• Simple answer: sign $h(m)$
• Requires collision-resistant hash-function $h$
• Idea: randomize before feeding $m$ into $h$
  • Pick random $r$
  • Compute $h(r \mid m)$
  • Send $r$ as part of the signature
• Make deterministic: $r \leftarrow \text{PRF}(s, m)$ for secret $s$
• Signature scheme is now collision resilient
Merkle, 1979: Leverage one-time signatures to multiple messages

Binary hash tree on top of OTS public keys
• Merkle, 1979: Leverage one-time signatures to multiple messages
• Binary hash tree on top of OTS public keys
- Use OTS keys sequentially
- \( \text{SIG} = (i, \text{sign}(M, X_i), Y_i, \text{Auth}) \)
- Signer needs to remember current index (\( \Rightarrow \) stateful scheme)
Merkle security

• Informally:
  • requires **EUF-CMA-secure** OTS
  • requires collision-resistant hash in the tree
• Can apply bitmask trick to get rid of collision-resistance assumption
• Merkle signatures are **stateful**
Keygen memory usage

• Keygen needs to compute the whole tree from leaves to root
• Naive implementation uses $\Theta(2^h)$ memory
Keygen memory usage

- Keygen needs to compute the whole tree from leaves to root
- Naive implementation uses $\Theta(2^h)$ memory
- Better approach, call `treehash` for each leaf, left to right:

```python
function treehash(stack, leaf node N)
    while stack.peek() is on the same level as N do
        neighbor ← stack.pop()
        N ← $H(neighbor \| N)$
    end while
    stack.push(N)
end function
```
• Keygen needs to compute the whole tree from leaves to root
• Naive implementation uses $\Theta(2^h)$ memory
• Better approach, call `treehash` for each leaf, left to right:

```java
function treehash(stack, leaf node N)
    while stack.peek() is on the same level as N do
        neighbor ← stack.pop()
        N ← H(neighbor || N)
    end while
    stack.push(N)
end function
```

• After going through all leaves, root will be on the top of the stack
• Memory requirement: $h + 1$ hashes
State size vs. signing speed

- KeyGen needs to compute the whole tree, but how about signing?

  - Can simply remember the tree from KeyGen: large secret key
  - Can recompute tree every time: very slow signing
  - Obvious tradeoff: remember last authentication path
  - Most of the time can reuse most nodes
  - Signing speed now depends largely on index
  - Idea: balance computations, store nodes required for future signatures
  - Commonly used algorithm (again allowing tradeoffs): BDS traversal
    Buchmann, Dahmen, Schneider, 2008: Merkle tree traversal revisited
State size vs. signing speed

- KeyGen needs to compute the whole tree, but how about signing?
- Can simply remember the tree from KeyGen: large secret key
State size vs. signing speed

- KeyGen needs to compute the whole tree, but how about signing?
- Can simply remember the tree from KeyGen: large secret key
- Can recompute tree every time: very slow signing
State size vs. signing speed

- KeyGen needs to compute the whole tree, but how about signing?
- Can simply remember the tree from KeyGen: large secret key
- Can recompute tree every time: very slow signing
- Obvious tradeoff: remember last authentication path
- Most of the time can reuse most nodes
State size vs. signing speed

- KeyGen needs to compute the whole tree, but how about signing?
- Can simply remember the tree from KeyGen: large secret key
- Can recompute tree every time: very slow signing
- Obvious tradeoff: remember last authentication path
- Most of the time can reuse most nodes
- Signing speed now depends largely on index
- Idea: balance computations, store nodes required for future signatures
- Commonly used algorithm (again allowing tradeoffs): **BDS traversal** Buchmann, Dahmen, Schneider, 2008: *Merkle tree traversal revisited*

Stateful signatures: downside

- Secret key changes with every signature
- Going back to previous secret key is security disaster
Stateful signatures: downside

- Secret key changes with every signature
- Going back to previous secret key is security disaster
- Huge problem in many contexts:
  - Backups
  - VM Snapshots
  - Load balancing
  - API is incompatible!
Stateful signatures: advantage

- Remember forward secrecy?: old ciphertexts remain secure after key compromise
- Signature **forward security**: old signatures remain valid after key compromise

For Hash-based signatures:
- generate OTS secret keys as $s_i = h(s_{i-1})$
- store only next valid OTS secret key
- Need to keep hashes of old public keys
- After key compromise publish index of compromised key
- Signatures with lower index remain valid
Stateful signatures: advantage

- Remember forward secrecy?: old ciphertexts remain secure after key compromise
- Signature **forward security**: old signatures remain valid after key compromise
- Need “timestamp” baked into signature
- Secret key has to evolve to disable signing “in the past”
Stateful signatures: advantage

• Remember forward secrecy?: old ciphertexts remain secure after key compromise
• Signature **forward security**: old signatures remain valid after key compromise
• Need “timestamp” baked into signature
• Secret key has to evolve to disable signing “in the past”
• For Hash-based signatures:
  • generate OTS secret keys as $s_i = h(s_{i-1})$
  • store only next valid OTS secret key
  • Need to keep hashes of old public keys
Stateful signatures: advantage

- Remember forward secrecy?: old ciphertexts remain secure after key compromise
- Signature **forward security**: old signatures remain valid after key compromise
- Need “timestamp” baked into signature
- Secret key has to evolve to disable signing “in the past”
- For Hash-based signatures:
  - generate OTS secret keys as $s_i = h(s_{i-1})$
  - store only next valid OTS secret key
  - Need to keep hashes of old public keys
- After key compromise publish index of compromised key
- Signatures with lower index remain valid
Multi-tree constructions

- Remember that KeyGen has to compute the whole tree
- Infeasible for very large trees
Multi-tree constructions

- Remember that KeyGen has to compute the whole tree
- Infeasible for very large trees
- Idea: generate all secret keys pseudo-randomly
- Use PRF on secret seed with position in the tree
• Remember that KeyGen has to compute the whole tree
• Infeasible for very large trees
• Idea: generate all secret keys pseudo-randomly
• Use PRF on secret seed with position in the tree
• Use hierarchy of trees, **connected via one-time signatures**
• Key generation computes only the top tree
• Many more size-speed tradeoffs

Daniel J. Bernstein
Daira Hopwood
Andreas Hülsing
Tanja Lange
Ruben Niederhagen
Louiza Papachristodoulou
Michael Schneider
Peter Schwabe
Zooko Wilcox-O’Hearn

Daniel J. Bernstein
Daira Hopwood
Andreas Hülsing
Tanja Lange
Ruben Niederhagen
Louiza Papachristodoulou
Michael Schneider
Peter Schwabe
Zooko Wilcox-O’Hearn
The SPHINCS approach

- Use a “hyper-tree” of total height $h$
- Parameter $d \geq 1$, such that $d \mid h$
- Each (Merkle) tree has height $h/d$
- $(h/d)$-ary certification tree
The SPHINCS approach

- Pick index (pseudo-)randomly
- Messages signed with few-time signature scheme
- Significantly reduce total tree height
- Require $Pr[r\text{-times Coll}] \cdot Pr[\text{Forgery after } r \text{ signatures}] = \text{negl}(n)$
The HORS few-time signature scheme

- Lamport signatures reveal half of the secret key with each signature
The HORS few-time signature scheme

- Lamport signatures reveal half of the secret key with each signature
- Idea in HORS:
  - use **much bigger** secret key
  - reveal only small portion
  - sign hash $g(m)$; attacker does not control output of $g$
  - attacker won’t have *enough* secret-key to forge
The HORS few-time signature scheme

- Lamport signatures reveal half of the secret key with each signature
- Idea in HORS:
  - use **much bigger** secret key
  - reveal only small portion
  - sign hash $g(m)$; attacker does not control output of $g$
  - attacker won’t have *enough* secret-key to forge
- Example parameters:
  - Generate $sk = (r_0, \ldots, r_{2^{16}})$
  - Compute public key $(h(r_0), \ldots, h(r_{2^{16}}))$
The HORS few-time signature scheme

- Lamport signatures reveal half of the secret key with each signature
- Idea in HORS:
  - use **much bigger** secret key
  - reveal only small portion
  - sign hash $g(m)$; attacker does not control output of $g$
  - attacker won’t have *enough* secret-key to forge
- Example parameters:
  - Generate $sk = (r_0, \ldots, r_{2^{16}})$
  - Compute public key $(h(r_0), \ldots, h(r_{2^{16}}))$
  - Sign 512-bit hash $g(m) = (g_0, \ldots, g_{31})$
  - Each $g_i \in 0, \ldots, 2^{16}$
The HORS few-time signature scheme

- Lamport signatures reveal half of the secret key with each signature
- Idea in HORS:
  - use **much bigger** secret key
  - reveal only small portion
  - sign hash $g(m)$; attacker does not control output of $g$
  - attacker won’t have *enough* secret-key to forge
- Example parameters:
  - Generate sk = $(r_0, \ldots, r_{2^{16}})$
  - Compute public key $(h(r_0), \ldots, h(r_{2^{16}}))$
  - Sign 512-bit hash $g(m) = (g_0, \ldots, g_{31})$
  - Each $g_i \in 0, \ldots, 2^{16}$
  - Signature is $(r_{g_0}, \ldots, r_{g_{31}})$
  - Signature reveals 32 out of 65536 secret-key values
  - Even after, say, 5 signatures, attacker does not know enough secret key to forge with non-negligible probability
The HORST few-time signature scheme

- Problem with HORS: 2 MB public key
- public key becomes part of signature in complete construction!
The HORST few-time signature scheme

• Problem with HORS: 2 MB public key
• public key becomes part of signature in complete construction!
• Idea:
  • build hash-tree on top of public-key chunks
  • use root of tree as new public key (32 bytes)
  • include authentication paths in signature
The HORST few-time signature scheme

- Problem with HORS: 2 MB public key
- public key becomes part of signature in complete construction!
- Idea:
  - build hash-tree on top of public-key chunks
  - use root of tree as new public key (32 bytes)
  - include authentication paths in signature
- Signature size (naïve):

\[
32 \cdot 32 + 32 \cdot 16 \cdot 32 = 17408 \text{ Bytes}
\]
The HORST few-time signature scheme

• Problem with HORS: 2 MB public key
• public key becomes part of signature in complete construction!
• Idea:
  • build hash-tree on top of public-key chunks
  • use root of tree as new public key (32 bytes)
  • include authentication paths in signature
• Signature size (naïve):

\[32 \cdot 32 + 32 \cdot 16 \cdot 32 = 17408 \text{ Bytes}\]

• Signature size (somewhat optimized): 13312 Bytes
• Designed for 128 bits of post-quantum security
• Support up to $2^{50}$ signatures
• 12 trees of height 5 each
• Designed for 128 bits of post-quantum security
• Support up to $2^{50}$ signatures
• 12 trees of height 5 each
• $n = 256$ bit hashes in WOTS and HORST
• Winternitz parameter $w = 16$
• HORST with $2^{16}$ expanded-secret-key chunks (total: 2 MB)
SPHINCS-256

- Designed for 128 bits of post-quantum security
- Support up to $2^{50}$ signatures
- 12 trees of height 5 each
- $n = 256$ bit hashes in WOTS and HORST
- Winternitz parameter $w = 16$
- HORST with $2^{16}$ expanded-secret-key chunks (total: 2 MB)
- $m = 512$ bit message hash (BLAKE-512)
- ChaCha12 as PRG
Cost of SPHINCS-256 signing

- Three main components:
  - PRG for HORST secret-key expansion to 2 MB
  - Hashing in WOTS and HORS public-key generation:
    \[ F : \{0, 1\}^{256} \rightarrow \{0, 1\}^{256} \]
  - Hashing in trees (mainly HORST public-key):
    \[ H : \{0, 1\}^{512} \rightarrow \{0, 1\}^{256} \]
- Overall: 451 456 invocations of \( F \), 91 251 invocations of \( H \)
Cost of SPHINCS-256 signing

• Three main components:
  • PRG for HORST secret-key expansion to 2 MB
  • Hashing in WOTS and HORS public-key generation:
    \[ F : \{0, 1\}^{256} \to \{0, 1\}^{256} \]
  • Hashing in trees (mainly HORST public-key):
    \[ H : \{0, 1\}^{512} \to \{0, 1\}^{256} \]

• Overall: 451 456 invocations of \( F \), 91 251 invocations of \( H \)

• Full hash function would be overkill for \( F \) and \( H \)

• Construction in SPHINCS-256:
  • \( F(M_1) = \text{Chop}_{256}(\pi(M_1||C)) \)
  • \( H(M_1||M_2) = \text{Chop}_{256}(\pi(\pi(M_1||C) \oplus (M_2||0^{256}))) \)
Cost of SPHINCS-256 signing

• Three main components:
  • PRG for HORST secret-key expansion to 2 MB
  • Hashing in WOTS and HORS public-key generation:
    \[ F : \{0, 1\}^{256} \rightarrow \{0, 1\}^{256} \]
  • Hashing in trees (mainly HORST public-key):
    \[ H : \{0, 1\}^{512} \rightarrow \{0, 1\}^{256} \]

• Overall: 451,456 invocations of \( F \), 91,251 invocations of \( H \)

• Full hash function would be overkill for \( F \) and \( H \)

• Construction in SPHINCS-256:
  • \( F(M_1) = \text{Chop}_{256}(\pi(M_1||C)) \)
  • \( H(M_1||M_2) = \text{Chop}_{256}(\pi(\pi(M_1||C) \oplus (M_2||0^{256}))) \)

• Use fast ChaCha12 permutation for \( \pi \)

• All building blocks (PRG, message hash, \( H, F \)) built from very similar permutations
SPHINCS-256 speed and sizes

SPHINCS-256 sizes

- ≈ 40 KB signature
- ≈ 1 KB public key (mainly bitmasks)
- ≈ 1 KB private key
SPHINCS-256 speed and sizes

**SPHINCS-256 sizes**

- $\approx 40$ KB signature
- $\approx 1$ KB public key (mainly bitmasks)
- $\approx 1$ KB private key

**High-speed implementation**

- Target Intel Haswell with 256-bit AVX2 vector instructions
- Use $8 \times$ parallel hashing, vectorize on high level
- $\approx 1.6$ cycles/byte for $H$ and $F$
SPHINCS-256 speed and sizes

SPHINCS-256 sizes

- ≈ 40 KB signature
- ≈ 1 KB public key (mainly bitmasks)
- ≈ 1 KB private key

High-speed implementation

- Target Intel Haswell with 256-bit AVX2 vector instructions
- Use $8 \times$ parallel hashing, vectorize on high level
- ≈ 1.6 cycles/byte for $H$ and $F$

SPHINCS-256 speed

- Signing: $< 52$ Mio. Haswell cycles ($> 200$ sigs/sec, 4 Core, 3GHz)
- Verification: $< 1.5$ Mio. Haswell cycles
- Keygen: $< 3.3$ Mio. Haswell cycles
• Remember tightness loss from many hash calls
• SPHINCS and SPHINCS\(^+\) have \textit{many} hash calls
• Remember tightness loss from many hash calls
• SPHINCS and SPHINCS\(^+\) have \textit{many} hash calls
• Think of it as attacker solving one out of many 2nd preimage challenges
• Trivial (pre-quantum) attack:
  • try all inputs of appropriate size
  • win if output matches \textit{any of the challenges}
• Remember tightness loss from many hash calls
• SPHINCS and SPHINCS$^+$ have many hash calls
• Think of it as attacker solving one out of many 2nd preimage challenges
• Trivial (pre-quantum) attack:
  • try all inputs of appropriate size
  • win if output matches any of the challenges
• Idea: use different hash function for each call
• Use address in the tree to pick hash function
• Remember tightness loss from many hash calls
• SPHINCS and SPHINCS$^+$ have many hash calls
• Think of it as attacker solving one out of many 2nd preimage challenges
• Trivial (pre-quantum) attack:
  • try all inputs of appropriate size
  • win if output matches any of the challenges
• Idea: use different hash function for each call
• Use address in the tree to pick hash function
• Proposed in 2016 by Hülsing, Rijneveld, and Song
• First adopted in XMSS (see RFC 8391)
• Remember tightness loss from many hash calls
• SPHINCS and SPHINCS\(^+\) have many hash calls
• Think of it as attacker solving one out of many 2nd preimage challenges
• Trivial (pre-quantum) attack:
  • try all inputs of appropriate size
  • win if output matches any of the challenges
• Idea: use different hash function for each call
• Use address in the tree to pick hash function
• Proposed in 2016 by Hülsing, Rijneveld, and Song
• First adopted in XMSS (see RFC 8391)
• Merge with random bitmasks into tweakable hash function
• NIST proposal: tweakable hash from SHA-256, SHAKE-256, or Haraka
From SPHINCS to SPHINCS\(^+\), part II

- Verifiable index computation:
  - SPHINCS:
    - \((i, r) \leftarrow \text{PRF}(s, m),\)
    - \(d \leftarrow h(r, m)\)
    - sign digest \(d\) with FTS
    - include \(i\) in signature
  - SPHINCS\(^+\):
    - \((i, r) \leftarrow \text{PRF}(s, m),\)
    - \(d \leftarrow h(r, m)\)
    - sign digest \(d\) with FTS
    - include \(i\) in signature
    - Verifier can check that \(d\) and \(i\) belong together
    - Attacker cannot pick \(d\) and \(i\) independently
  - Additionally: Improvements to FTS (FORS)
    - Use multiple smaller trees instead of one big tree
    - Per signature, reveal one secret-key leaf per tree
• Verifiable index computation:
  • SPHINCS:
    • \((i, r) \leftarrow \text{PRF}(s, m)\),
    • \(d \leftarrow h(r, m)\)
    • sign digest \(d\) with FTS
    • include \(i\) in signature
  • SPHINCS\(^+\):  
    • \(r \leftarrow \text{PRF}(s, m)\) 
    • \((i, d) \leftarrow h(r, m)\),
    • sign digest \(d\) with FTS
    • include \(r\) in signature
From SPHINCS to SPHINCS\(^+\), part II

- Verifiable index computation:
  - **SPHINCS:**
    - \((i, r) \leftarrow \text{PRF}(s, m)\),
    - \(d \leftarrow h(r, m)\)
    - sign digest \(d\) with FTS
    - include \(i\) in signature
  - **SPHINCS\(^+\):**
    - \(r \leftarrow \text{PRF}(s, m)\)
    - \((i, d) \leftarrow h(r, m)\),
    - sign digest \(d\) with FTS
    - include \(r\) in signature

- Verifier can check that \(d\) and \(i\) belong together
- Attacker cannot pick \(d\) and \(i\) independently
• Verifiable index computation:
  • SPHINCS:
    • \((i, r) \leftarrow \text{PRF}(s, m)\),
    • \(d \leftarrow h(r, m)\)
    • sign digest \(d\) with FTS
    • include \(i\) in signature
  • SPHINCS\(^+\):
    • \(r \leftarrow \text{PRF}(s, m)\)
    • \((i, d) \leftarrow h(r, m)\),
    • sign digest \(d\) with FTS
    • include \(r\) in signature

  • Verifier can check that \(d\) and \(i\) belong together
  • Attacker cannot pick \(d\) and \(i\) independently

  • Additionally: Improvements to FTS (FORS)
  • Use multiple smaller trees instead of one big tree
  • Per signature, reveal one secret-key leaf per tree
Know more?

https://sphincs.org