Post-quantum crypto on embedded microcontrollers

Peter Schwabe
peter@cryptojedi.org
https://cryptojedi.org
December 4, 2019
5 building blocks for a “secure channel”

**Symmetric crypto**

- Block or stream cipher (e.g., AES, ChaCha20)
- Authenticator (e.g., HMAC, GMAC, Poly1305)
- Hash function (e.g., SHA-2, SHA-3)
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- Signatures (e.g., RSA, DSA, ECDSA, EdDSA)
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**The asymmetric monoculture**

- All widely deployed asymmetric crypto relies on
  - the **hardness of factoring**, or
  - the **hardness of (elliptic-curve) discrete logarithms**
Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer

Peter W. Shor

Abstract

A digital computer is generally believed to be an efficient universal computing device; that is, it is believed able to simulate any physical computing device with an increase in computation time by at most a polynomial factor. This may not be true when quantum mechanics is taken into consideration. This paper considers factoring integers and finding discrete logarithms, two problems which are generally thought to be hard on a classical computer and which have been used as the basis of several proposed cryptosystems. Efficient randomized algorithms are given for these two problems on a hypothetical quantum computer. These algorithms take a number of steps polynomial in the input size, e.g., the number of digits of the integer to be factored.
“In the past, people have said, maybe it’s 50 years away, it’s a dream, maybe it’ll happen sometime. I used to think it was 50. Now I’m thinking like it’s 15 or a little more. It’s within reach. It’s within our lifetime. It’s going to happen.”

—Mark Ketchen (IBM), Feb. 2012, about quantum computers
Definition
Post-quantum crypto is (asymmetric) crypto that resists attacks using classical and quantum computers.
Post-quantum crypto

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5 main directions

- Lattice-based crypto (PKE and Sigs)
- Code-based crypto (mainly PKE)
- Multivariate-based crypto (mainly Sigs)
- Hash-based signatures (only Sigs)
- Isogeny-based crypto (so far, mainly PKE)
The NIST competition

Overview tweeted by Jacob Alperin-Sheriff on Dec 4, 2017.

<table>
<thead>
<tr>
<th>Count of Problem Category</th>
<th>Column Labels</th>
<th>Signature</th>
<th>Grand Total</th>
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<td>Row Labels</td>
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<td>Codes</td>
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<tr>
<td>RSA</td>
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<td>1</td>
<td>2</td>
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</table>

**Grand Total**

|               | 57 | 23 | 80 |
"Key exchange"

- What is meant is **key encapsulation mechanisms** (KEMs)
  - \((vk, sk) \leftarrow \text{KeyGen}()\)
  - \((c, k) \leftarrow \text{Encaps}(vk)\)
  - \(k \leftarrow \text{Decaps}(c, sk)\)

Status of the NIST competition

- In total 69 submissions accepted as "complete and proper"
- Several broken, 5 withdrawn
- Jan 2019: NIST announces 26 round-2 candidates
  - 17 KEMs and PKEs
  - 9 signature schemes
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“Performance (hardware+software) will play more of a role”

—Dustin Moody, May 2019
Round-2 of the NIST PQC project

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“...we will recommend that teams generally focus their hardware implementation efforts on Cortex-M4”
—Daniel Apon, Feb 2019
Joint work with
Mathias Kannwischer, Joost Rijneveld, and Ko Stoffelen.

- Started as part of PQCRYPTO H2020 project
- Continued within EPOQUE ERC StG
- Library and testing/benchmarking framework
  - PQ-crypto on ARM Cortex-M4
  - Uses STM32F4 Discovery board
  - 192 KB of RAM, benchmarks at 24 MHz
- Easy to add schemes using NIST API
- Optimized SHA3 and AES shared across primitives
pqm4 usage

- Run functional tests of all primitives and implementations:
  python3 test.py
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- Run speed and stack benchmarks:
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- Generate testvectors, compare for consistency (also with host):
  
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- Run speed and stack benchmarks:
  
  python3 benchmarks.py

- Easy to evaluate only subset of schemes, e.g.:

  python3 test.py newhope1024cca sphincs-shake256-128s
## Signatures (not) in pqm4

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Reference</th>
<th>Optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRYS{ALS}-Dilithium</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>FALCON</td>
<td>✓\textsubscript{RAM}</td>
<td>✓</td>
</tr>
<tr>
<td>GeMSS</td>
<td>✓\textsubscript{Key}</td>
<td>—</td>
</tr>
<tr>
<td>LUOV</td>
<td>✓</td>
<td>—</td>
</tr>
<tr>
<td>MQDSS</td>
<td>✓\textsubscript{RAM}</td>
<td>—</td>
</tr>
<tr>
<td>Picnic</td>
<td>✓\textsubscript{RAM}</td>
<td>—</td>
</tr>
<tr>
<td>qTESLA</td>
<td>✓</td>
<td>—</td>
</tr>
<tr>
<td>Rainbow</td>
<td>✓\textsubscript{Key}</td>
<td>—</td>
</tr>
<tr>
<td>SPHINCS+</td>
<td>✓</td>
<td>—</td>
</tr>
</tbody>
</table>

\textbf{\textit{X}}_{\text{Key}}: keys too large \quad \textbf{\textit{X}}_{\text{RAM}}: implementation uses too much RAM

\textbf{\textit{X}}_{\text{Lib}}: available implementations depend on external libraries
# KEMs (not) in pqm4

<table>
<thead>
<tr>
<th>KEM</th>
<th>reference</th>
<th>optimized</th>
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</thead>
<tbody>
<tr>
<td>BIKE</td>
<td>$\times_{Lib}$</td>
<td>—</td>
</tr>
<tr>
<td>Classic McEliece</td>
<td>$\times_{Key}$</td>
<td>—</td>
</tr>
<tr>
<td>CRYSSTALS-Kyber</td>
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<td>✓</td>
</tr>
<tr>
<td>Frodo-KEM</td>
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<td>✓</td>
</tr>
<tr>
<td>HQC</td>
<td>$\times_{Lib}$</td>
<td>—</td>
</tr>
<tr>
<td>LAC</td>
<td>✓</td>
<td>—</td>
</tr>
<tr>
<td>LEDAcrypt</td>
<td>$\times_{RAM}$</td>
<td>WIP</td>
</tr>
<tr>
<td>NewHope</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>NTRU</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>NTRU Prime</td>
<td>✓</td>
<td>—</td>
</tr>
<tr>
<td>NTS-KEM</td>
<td>$\times_{Key}$</td>
<td>—</td>
</tr>
<tr>
<td>ROLLO</td>
<td>$\times_{Lib}$</td>
<td>—</td>
</tr>
<tr>
<td>Round5</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>RQC</td>
<td>$\times_{Lib}$</td>
<td>—</td>
</tr>
<tr>
<td>SABER</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>SIKE</td>
<td>✓</td>
<td>—</td>
</tr>
<tr>
<td>ThreeBears</td>
<td>✓</td>
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<td>—</td>
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<td>—</td>
</tr>
<tr>
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<td>✓</td>
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Learning with errors (LWE)

- Given uniform $A \in \mathbb{Z}_q^{k \times \ell}$
- Given “noise distribution” $\chi$
- Given samples $As + e$, with $e \leftarrow \chi$
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- Structured lattices: work in $\mathbb{Z}_q[x]/f$
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- Given uniform $A \in \mathbb{Z}_q^{k \times \ell}$
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Lattice-based KEMs – the basic idea

<table>
<thead>
<tr>
<th>Alice (server)</th>
<th>Bob (client)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s, e \leftarrow \chi$</td>
<td>$s', e' \leftarrow \chi$</td>
</tr>
<tr>
<td>$b \leftarrow as + e$</td>
<td>$b' \rightarrow u$</td>
</tr>
<tr>
<td>$u \leftarrow as' + e'$</td>
<td>$v' = bs'$</td>
</tr>
</tbody>
</table>

Alice has $v = us = ass' + e's$

Bob has $v' = bs' = ass' + es'$

- Secret and noise $s, s', e, e'$ are small
- $v$ and $v'$ are *approximately* the same
Core operation: multiplication in $\mathcal{R}_q = \mathbb{Z}_q[X]/f$

Power-of-two $q$

- Several schemes use $q = 2^m$, for small $m$
- Examples: Round5, NTRU, Saber
- More round-1 examples: Kindi, RLizard
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Prime “NTT-friendly” $q$

- Kyber and NewHope use prime $q$ supporting fast NTT
- For $A, B \in \mathcal{R}_q$, $A \cdot B = \text{NTT}^{-1}(\text{NTT}(A) \circ \text{NTT}(B))$
- NTT is Fourier Transform over finite field
- Use $f = X^n + 1$ for power-of-two $n$
Multiplication in $\mathbb{Z}_{2^m}[X]$

- Joint work with Matthias Kannwischer and Joost Rijneveld
- Represent coefficients as 16-bit integers
- No modular reductions required, $2^{16}$ is a multiple of $q = 2^m$
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- Schoolbook multiplication takes $n^2$ integer muls, $(n - 1)^2$ adds
- Can do better using Karatsuba:

\[
(a_\ell + X^k a_h) \cdot (b_\ell + X^k b_h) = a_\ell b_\ell + X^k(a_\ell b_h + a_h b_\ell) + X^n a_h b_h
\]

\[
= a_\ell b_\ell + X^k((a_\ell + a_h)(b_\ell + b_h) - a_\ell b_\ell - a_h b_h) + X^n a_h b_h
\]

- Recursive application yields complexity $\Theta(n^{\log_2 3})$
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- Recursive application yields complexity $\Theta(n^{\log_2 3})$
- Generalization: Toom-Cook
  - Toom-3: split into 5 multiplications of $1/3$ size
  - Toom-4: split into 7 multiplications of $1/4$ size
- Approach: Evaluate, multiply, interpolate
Initial observations

- Karatsuba/Toom is asymptotically faster, but isn’t for “small” polynomials
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- Toom-3 needs division by 2, loses 1 bit of precision
- Toom-4 needs division by 8, loses 3 bits of precision
- This limits recursive application when using 16-bit integers
- Can use Toom-4 only for $q \leq 2^{13}$
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  • Optimize Saber, $q = 2^{13}$, $n = 256$
  • Use Toom-4 + two levels of Karatsuba
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• Is this the best approach? How about other values of $q$ and $n$?
OPTIMIZE

ALL THE MULTIPLICATIONS!
Our approach

• Generate optimized assembly for Karatsuba/Toom
• Use Python scripts, receive as input $n$ and $q$
• Hand-optimize “small” schoolbook multiplications
  • Make heavy use of DSP “vector instructions”
  • Perform two $16 \times 16$-bit multiply-accumulate in one cycle
  • Carefully schedule instructions to minimize loads/stores
• Benchmark different options, pick fastest
## Multiplication results

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Approach</th>
<th>“small”</th>
<th>Cycles</th>
<th>Stack</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Saber</strong> ((n = 256, q = 2^{13})**</td>
<td>Karatsuba only</td>
<td>16</td>
<td>41 121</td>
<td>2 020</td>
</tr>
<tr>
<td></td>
<td>Toom-3</td>
<td>11</td>
<td>41 225</td>
<td>3 480</td>
</tr>
<tr>
<td><strong>Toom-4</strong></td>
<td></td>
<td>16</td>
<td>39 124</td>
<td>3 800</td>
</tr>
<tr>
<td></td>
<td>Toom-4 + Toom-3</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Kindi-256-3-4-2</strong> ((n = 256, q = 2^{14})**</td>
<td>Karatsuba only</td>
<td>16</td>
<td>41 121</td>
<td>2 020</td>
</tr>
<tr>
<td></td>
<td>Toom-3</td>
<td>11</td>
<td>41 225</td>
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<td></td>
<td>Toom-4</td>
<td>-</td>
<td>-</td>
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</tr>
<tr>
<td></td>
<td>Toom-4 + Toom-3</td>
<td>-</td>
<td>-</td>
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<tr>
<td><strong>NTRU-HRSS</strong> ((n = 701, q = 2^{13})**</td>
<td>Karatsuba only</td>
<td>11</td>
<td>230 132</td>
<td>5 676</td>
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<tr>
<td></td>
<td>Toom-3</td>
<td>15</td>
<td>217 436</td>
<td>9 384</td>
</tr>
<tr>
<td><strong>Toom-4</strong></td>
<td></td>
<td>11</td>
<td>182 129</td>
<td>10 596</td>
</tr>
<tr>
<td></td>
<td>Toom-4 + Toom-3</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>NTRU-KEM-743</strong> ((n = 743, q = 2^{11})**</td>
<td>Karatsuba only</td>
<td>12</td>
<td>247 489</td>
<td>6 012</td>
</tr>
<tr>
<td></td>
<td>Toom-3</td>
<td>16</td>
<td>219 061</td>
<td>9 920</td>
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<tr>
<td><strong>Toom-4</strong></td>
<td></td>
<td>12</td>
<td>196 940</td>
<td>11 208</td>
</tr>
<tr>
<td></td>
<td>Toom-4 + Toom-3</td>
<td>16</td>
<td>197 227</td>
<td>12 152</td>
</tr>
<tr>
<td><strong>RLizard-1024</strong> ((n = 1024, q = 2^{11})**</td>
<td>Karatsuba only</td>
<td>16</td>
<td>400 810</td>
<td>8 188</td>
</tr>
<tr>
<td></td>
<td>Toom-3</td>
<td>11</td>
<td>360 589</td>
<td>13 756</td>
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NTT-based multiplication

- Joint work with Leon Botros and Matthias Kannwischer
- Primary goal: optimize Kyber
- Secondary effect: optimize NewHope (improved by Gérard)
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- Evaluate polynomial \( f = f_0 + f_1 X + \cdots + f_{n-1} X^{n-1} \) at all \( n \)-th roots of unity
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  - $f_0$ has $n/2$ coefficients
  - Evaluate $f_0$ at all $(n/2)$-th roots of unity by recursive application
  - Same for $f_1$
• First thing to do: replace recursion by iteration
• Loop over \( \log n \) levels with \( n/2 \) “butterflies” each
NTT-based multiplication

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- Butterfly on level $k$:
  - Pick up $f_i$ and $f_{i+2^k}$
  - Multiply $f_{i+2^k}$ by a power of $\omega$ to obtain $t$
  - Compute $f_{i+2^k} \leftarrow a_i - t$
  - Compute $f_i \leftarrow a_i + t$

Main optimizations on Cortex-M4:
- “Merge” levels: fewer loads/stores
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<th>Encapsulation</th>
<th>Decapsulation</th>
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**Comparison:** Curve25519 scalarmult: 625 358 cycles
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- For FALCON more speedups possible using floating-point arithmetic
Resources online

- NIST PQC website:
  https://csrc.nist.gov/Projects/Post-Quantum-Cryptography

- NIST “PQC forum” mailing list:
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- pqm4 library and benchmarking suite:
  https://github.com/mupq/pqm4

- Code of $\mathbb{Z}_{2^m}[x]$ multiplication paper, including scripts:
  https://github.com/mupq/polymul-z2mx-m4

- $\mathbb{Z}_{2^m}[x]$ multiplication paper:
  https://cryptojedi.org/papers/#latticem4

- Kyber/NTT optimization paper:
  https://cryptojedi.org/papers/#nttm4